Completely Normal and Weak Completely Normal in Intuitionistic Topological Spaces

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Abstract - In this paper we introduce completely normal in intuitionistic topological spaces and study the relation among them .Also we introduce a weak completely normal in intuitionistic topological spaces and study the relation among them . Finally we study the relation between completely normal and weak completely normal in intuitionistic topological spaces .

Keys words- intuitionistic topological spaces, completely normal, weak completely normal.

1- Introduction

The concept of fuzzy set was introduced by Zadeh [15] in his classical paper 1965. After the discovery of the fuzzy sets much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a subset naturally plays a very significant role in the study of fuzzy topology which introduced by Chang 1968 [6] ,and later by Malghan and Benchalli in 1981 [10] . In 1983, Atanassov introduced the concept of " Intuitionistic fuzzy set " [1],[2],[3],[4] using a type of generalized fuzzy set , Later, the concept is used to define intuitionistic fuzzy special sets by Coker [7], and intuitionistic fuzzy topological spaces are introduced by Coker [8]. In this direction, the concept of separation axioms in intuitionistic fuzzy topological spaces which introduced by Bayhan , S . and Coker, D [5] . Also concept of intuitionistic topological spaces which introduced by Coker in 2000 [9] .In this paper we introduce completely normal in intuitionistic topological spaces and study the relation among them .Also we introduce a weak completely normal in intuitionistic topological spaces and study the relation among them .Finally we study the relation between completely normal and weak completely normal in intuitionistic topological spaces .

Preliminaries

Definition 1.1 [7]
Let X be a non-empty set . An intuitionistic set A is an object having the form $A=\langle x,A_1,A_2\rangle$,where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2=\emptyset$. The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A .

Remark
Any subset A of X can be regarded as intuitionistic set having the form $A=\langle x,A,A^c\rangle$.

Definition 1. 2 [7]
Let X be a nonempty set , and let $A=\langle x,A_1,A_2\rangle$ and $B=\langle x,B_1,B_2\rangle$ be intuitionistic sets respectively, furthermore, let $\{A_i; i\in J\}$ be an arbitrary family of intuitionistic sets in X , where $A_i=\langle x,A_i^{(1)},A_i^{(2)}\rangle$, then $A\subseteq B$ if and only if $A_1\subseteq B_1$ and $B_2\subseteq A_2$, A = B if and only if $A\subseteq B$ and $B\subseteq A$, The complement of A is denoted by $\overline{A}$ and defined by $\overline{A}=\langle x,A_2,A_1\rangle$, $FA=\langle x,A_1,A_1^c\rangle$, $SA=\langle x,A_2^c,A_2\rangle$, $\bigcup A_i=\langle x,\bigcup A_i^{(1)},\bigcap A_i^{(2)}\rangle$, $\bigcap A_i=\langle x,\bigcap A_i^{(1)},\bigcup A_i^{(2)}\rangle$, $\tilde{\phi}=\langle x,\phi,X\rangle$, $\tilde{X}=\langle x,X,\phi\rangle$.

Definition 1.3 [7]
Let X be a nonempty set , $p\in X$ a fixed element in X , and let $A=\langle x,A_1,A_2\rangle$ be an intuitionistic set (IS, for short).The IS $\dot{p}$ defined by $\dot{p}=\langle x,\{p\},\{p\}^c\rangle$ is called an intuitionistic point (IP for short) in X .The IS $\ddot{p}=\langle x,\phi,\{p\}^c\rangle$ is called a vanishing intuitionistic point (VIP, for short) in X .The IS $\dot{p}$ is said to be contained in A.($\dot{p}\in A$, for short) if and only if $p\in A_1$,and similarly IS $\ddot{p}$ contained in A. ($\ddot{p}\in A$, for short) if and only if $p\notin A_2$. For a given IS A in X , we may write $A=\left(\bigcup\{\dot{p}:\dot{p}\in A\}\right)\bigcup\left(\bigcup\{\ddot{p}:\ddot{p}\in A\}\right)$,and whenever A is not a proper IS (i.e., if A is not of the form $A=\langle x,A_1,A_2\rangle$ (where $A_1\bigcup A_2\notin X$), then $A=\left(\bigcup\{\dot{p}:\dot{p}\in A\}\right)$hold . In general, any IS A in X can be written in the form $A=\dot{A}\bigcup\ddot{A}$ where $\dot{A}=\bigcup\{\dot{p}:\dot{p}\in A\}$ and $\ddot{A}=\bigcup\{\ddot{p}:\ddot{p}\in A\}$.

Definition 1. 4 [7]
Let X and Y be two nonempty sets and $f:X\to Y$ be a function.

a) If $B=\langle y,B_1,B_2\rangle$ is an IS in Y, then the preimage (inverse image) of B under f is denoted by $f^{-1}(B)$ is an IS in X and defined by $f^{-1}(B)=\langle x,f^{-1}(B_1),f^{-1}(B_2)\rangle$.

b) If $A=\langle x,A_1,A_2\rangle$ is an IS in X, then the image of A under f denoted by $f(A)$ is an IS in Y defined by $f(A)=\langle y,f(A_1),f(A_2)\rangle$, where $f(A)=\left(f\left(A_2^c\right)\right)^c$.

Definition 1.5 [7],[8]
An intuitionistic topology on a nonempty set X is a family T of an intuitionistic sets in X satisfying the following conditions.

(1) $\tilde{\phi},\tilde{X}\in T$.
(2) T is closed under finite intersections.
(3) T is closed under arbitrary unions.

The pair (X,T) is called an intuitionistic topological space (ITS,for short) . Any element in T is usually called intuitionistic open set (IOS for short).The complement of an IOS in a ITS (X,T) is called intuitionistic closed set (ICS, for short).

Definition 1. 6 [14]

Let (X,T) be an ITS and let $A=\langle x,A_1,A_2\rangle$ be an intuitionistic subset (IS's, for short). in a set X . The interior(IntA, for short). and closure(ClA, for short). of a set A of X are defined : $\operatorname{int} A=\bigcup\{G:G\subseteq A,G\in T\}$, $ClA=\bigcap\{F:A\subseteq F,\overline{F}\in T\}$ .In other words: The intA is the largest intuitionistic open set contained in A, and ClA is the smallest intuitionistic closed set contain A i.e., $\operatorname{int} A\subseteq A$ and $A\subseteq ClA$. In the following definition we give a product of an intuitionistic set and a product of an intuitionistic topological space .

Definition 1. 7 [5] ,[14]

Giving the nonempty set X, we define the diagonal Δ_x as IS in X×X in the following way:

$\Delta_x\langle(x_1,x_2),\{(x_1,x_2):x_1=x_2\},\{(x_1,x_2):x_1\neq x_2\}\rangle$.

Now we are ready to give the definition of IP and VIP of the product X×Y .

Definition 1.8. [5]

Let X and Y be two nonempty set's and $(p,q)\in X\times Y$ be a fixed element in X ×Y, then the IP (p,q) is contained in U ×V($(p,q)U\times V$,for short) if and only if $(p,q)\in U_1\bigcup V_1$, and IVP $(\ddot{p},\ddot{q})$ is contained in U×V ($(\ddot{p},\ddot{q})U\times V$,for short) if and only if $(p,q)\notin(U_2^c\times V_2^c)$,or equivalently $(p,q)\in(U_2^c\times V_2^c)$

Definition 1.9.[12]

Let (X,T) be an ITS, then (X,T) is said to be :

a)R(i) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{x}\notin F$ there exists $U,V\in T$ such that $\tilde{x}\notin U,F\subseteq V$ and $U\bigcap V=\phi$.

b) R(ii) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{\tilde{x}}\notin F$ there exists $U,V\in T$ such that $\tilde{\tilde{x}}\notin U,F\subseteq V$ and $U\bigcap V=\phi$.

c) R(iii) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{x}\in F$ there exists $U,V\in T$ such that $\tilde{x}\in U,F\subseteq V$ and $U\subseteq\overline{V}$.

d) R(iv) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{\tilde{x}}\in F$ there exists $U,V\in T$ such that $\tilde{\tilde{x}}\in U,F\subseteq V$ and $U\subseteq\overline{V}$.

Definition 1.10.[12]

Let (X,T) be an ITS, then (X,T) is said to be :

a) wR(i) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{x}\in F$ there exists $U,V\in WOX$ such that $\tilde{x}\in U,F\subseteq V$ and $U\cap V=\tilde{\phi}$. Where WO(X) is set of all weak intuitionistic open set in X .

b) wR(ii) if and only if for each $x\in X$ and $F\subseteq X,F$ is ICS and $\tilde{\tilde{x}}\notin F$ there exists $U,V\in WOX$ such that $\tilde{\tilde{x}}\in U,F\subseteq V$ and $U\bigcap V=\tilde{\phi}$. Where WO(X) is set of all weak intuitionistic open set in X .

SECTION.2 COMPLETELY NORMAL IN INTUITIONISTIC TOPOLOGICAL SPACES

In this paper we introduce completely normal in intuitionistic topological spaces and study the relation among them .Also we introduce a weak completely normal in intuitionistic topological spaces and study the relation among them .

Defintion 2-1

Let (X,T) be an ITS, and let $E\subseteq X$ .We say two sets F, G are separation for set E iff satisfies the following conditions $F,G\neq\tilde{\phi}.E=F\cup G.(F\cap\overline{G})\cup(G\cap\overline{F})=\phi$

Definition 2-2

Let (X,T) be an ITS, then (X,T) is said to be :

a) CN(i) if and only if for each $x\in X$, $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\cap V=\tilde{\phi}$.

b) CN(ii) if and only if for each $x\in X$, $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{\tilde{x}}\in F\subseteq U,\tilde{\tilde{x}}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$.

c) CN(iii) if and only if for each $x\in X$ $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F\subseteq U,\tilde{x}\notin G\subseteq V$ and $U\subseteq\overline{V}$.

d) CN(iv) if and only if for each $x\in X$, $E\subseteq X$ and F , G are separation for E there exists $U,V\in T$ such that $\tilde{\tilde{x}}\in F\subseteq U,\tilde{\tilde{x}}\notin G\subseteq V$ and $U\subseteq\overline{V}$.

Proposition 2-3

Let (X,T) be an ITS, then the following implications are valid:

Proof: CN(i) → CN(ii)

Let (X,T) be satisfies CN(i)if for each $x\in X$, $E\subseteq X$, and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\cap V=\tilde{\phi}$. $\tilde{x}\in F\subseteq U$ iff $x\in F_1$ and since $U\cap V=\tilde{\phi}$ so that $x\notin G$ iff $\tilde{\tilde{x}}\in F\subseteq U$. Similarly If $\tilde{x}\notin G\subseteq V$, we get $\tilde{\tilde{x}}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$ .Therefore (X,T) be satisfies CN(ii) .

CN(i) → CN(iii)

Let (X,T) be satisfies CN(i) if for each $x\in X$, $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\cap V=\tilde{\phi}$, $\tilde{x}\in F,F\subseteq U$ iff $\tilde{x}\notin G,G\subseteq V$ and since $U\cap V=\tilde{\phi}$,so that $\tilde{x}\notin G,\overline{G}\subseteq\overline{V}$. Therefore $U\subseteq\overline{V}$ .Similarly If $\tilde{x}\notin G,G\subseteq V$, we get and $U\subseteq\overline{V}$ .Therefore (X,T) be satisfies CN(iii).

CN(ii) → CN(iv)

Let (X,T) be satisfies CN(ii) if for each, $x\in X$, $E\subseteq X$ and F,G are separation for E there exist $U,V\in T$ such that $\tilde{\tilde{x}}\in F\subseteq U$, $\tilde{\tilde{x}}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$. $\tilde{\tilde{x}}\in F\subseteq U$ iff $\tilde{\tilde{x}}\notin G,G\subseteq V$, since $U\cap V=\tilde{\phi}$ .This implies $\tilde{x}\notin G,\overline{G}\subseteq\overline{V}$, therefore $U\subseteq\overline{V}$ .Similarly If $\tilde{\tilde{x}}\notin F\subseteq V$,we get $\tilde{\tilde{x}}\notin G\subseteq V$ and $U\subseteq\overline{V}$. Therefore (X,T) be satisfies CN(iv)

CN(iii) → CN(vi)

Let (X,T) be satisfies CN(iii) if for each $x\in X$, $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\subseteq\overline{V}$. $\tilde{x}\in F,F\subseteq U$ iff $x\in F_1\subseteq U$ since $\langle x,F_1,F_2\rangle$ and $F_1\cap F_2=\tilde{\phi}$,this implies $x\notin F_2$ iff $\tilde{\tilde{x}}\in F\subseteq U$ and $U\subseteq\overline{V}$ .Similarly If $\tilde{x}\notin G,G\subseteq V$ and

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$U \subseteq \bar{V}$, we get $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$.
In general the converse of the diagram appears in Proposition 2-3 is not true in general. The following counter examples show the cases.

Example 2-4: Let $X = \{a,b\}$ and define $T = \{\tilde{\phi}, \tilde{X}, A, B, C\}$ where $A = \langle x,\{b\},\{a\}\rangle$, $B = \langle x,\phi,\{b\}\rangle$, $C = \langle x,\{b\},\phi\rangle$. Then (X,T) satisfies CN(ii) because for each $x \in X$, $E \subseteq X$, $F = \langle x,\{b\},\{a\}\rangle$ $G = \langle x,\phi,\{b\}\rangle$ are separation for E there exists $A, B \in T$ such that $\tilde{\tilde{a}} \in F \subseteq A, \tilde{\tilde{a}} \notin G \subseteq B$, and $A \cap B = \tilde{\phi}$ .Also $\tilde{\tilde{b}} \in G \subseteq B, \tilde{\tilde{b}} \notin F \subseteq A$ $\tilde{b} \in G \subseteq B$, and $A \cap B = \phi$,But (X,T) is not satisfies CN(i),because there is no exist two open sets satisfies condition of CN(i).

Example 2-5: Let $X = \{a,b\}$ and define $T = \{\tilde{\phi}, \tilde{X}, A, B, C, D, E, F\}$,where $A = \langle x,\{a\},\phi\rangle$, $B = \langle x,\{b\},\{a\}\rangle$, $C = \langle x,\phi,\phi\rangle$. $D = \langle x,\phi,\{a\}\rangle$, $E = \langle x,\phi,\{b\}\rangle$, $F = \langle x,\{b\},\phi\rangle$. Then (X,T) satisfies CN(iii) because for each $x \in X$, $E \subseteq X$ and $H = \langle x,\{a\},\{b\}\rangle$, $G = \langle x,\{b\},\phi\rangle$ are separation for E there exists $A, B \in T$ such that $\tilde{a} \in H \subseteq A, \tilde{a} \notin G \subseteq D$ and $A \subseteq \bar{D}$ .Also $\tilde{b} \in G \subseteq F, \tilde{b} \notin E \subseteq E$ and $F \subseteq \bar{E}$. But (X,T) is not satisfies CN(i),because there is no exist two open sets satisfies condition of CN(i)

Example 2-6: Let $X = \{a,b\}$ and define $T = \{\tilde{\phi}, \tilde{X}, A, B, C\}$, where $A = \langle x,\phi,\{a\}\rangle$, $B = \langle x,\phi,\{b\}\rangle$, $C = \langle x,\phi,\phi\rangle$. Then (X,T) satisfies CN(iv),because for each $x \in X$, $E \subseteq X$ and $F = \langle x,\phi,\{a\}\rangle$ $G = \langle x,\phi,\{b\}\rangle$,are separation for E there exists $A, B \in T$ such that $\tilde{\tilde{a}} \in F \subseteq A, \tilde{\tilde{a}} \notin G \subseteq B$ and $A \subseteq \bar{B}$. Also $\tilde{\tilde{b}} \in G \subseteq B, \tilde{\tilde{b}} \notin F \subseteq A$ and $B \subseteq \bar{A}$.But (X,T) is not satisfies CN(iii),because there is no exist two open sets satisfies condition of CN(iii).Also (X,T) is not satisfies CN(ii),because there is no exist two open sets satisfies condition of CN(ii) .

Definition 2-7

Let (X,T) be an ITS, then (X,T) is said to be :

a) WCN(i) if and only if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F, F \subseteq U$, $\tilde{\tilde{x}} \notin G, G \subseteq V$ and $U \cap V = \tilde{\phi}$.

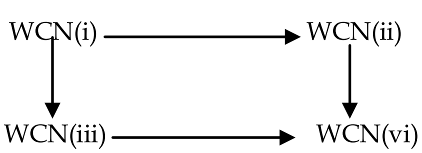
b) WCN(ii) if and only if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F \subseteq U, \tilde{\tilde{x}} \notin G \subseteq V$ and $U \cap V = \tilde{\phi}$.

c) WCN(iii) if and only if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F, F \subseteq U$, $\tilde{\tilde{x}} \notin G, G \subseteq V$ and $U \subseteq \bar{V}$.

d) WCN(iv) if and only if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F \subseteq U, \tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$.

Proposition 2-8

Let (X,T) be an ITS, then the following implications are valid:



Proof : WCN(i) $\longrightarrow$ WCN(ii)

Let (X,T) be satisfies WCN(i) if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F, F \subseteq U$, $\tilde{\tilde{x}} \notin G, G \subseteq V$ and $U \cap V = \tilde{\phi}$. $\tilde{\tilde{x}} \in F, F \subseteq U$ iff $x \in F_1 \subseteq U$ and since $U \cap V = \tilde{\phi}$ so that $\tilde{\tilde{x}} \notin G \subseteq V$ iff $\tilde{\tilde{x}} \in F \subseteq U$, . Similarly If $\tilde{\tilde{x}} \notin G \subseteq V$, we get $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \cap V = \tilde{\phi}$ .Therefore (X,T) be satisfies WCN(i).

WCN(i) $\longrightarrow$ WCN(iii)

Let (X,T) be satisfies WCN(i) if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F, F \subseteq U$, $\tilde{\tilde{x}} \notin G, G \subseteq V$ and $U \cap V = \tilde{\phi}$. $\tilde{\tilde{x}} \in F, F \subseteq U$ iff $\tilde{\tilde{x}} \notin G \subseteq V$ and since $U \cap V = \tilde{\phi}$,so that $\tilde{\tilde{x}} \in \bar{G} \subseteq \bar{V}$. Therefore $U \subseteq \bar{V}$ .Similarly If $\tilde{\tilde{x}} \notin G \subseteq V$, we get $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$ .Therefore (X,T) be satisfies WCN(iii).

WCN(ii) $\longrightarrow$ WCN(iv)

Let (X,T) be satisfies WCN(ii)if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exist $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F \subseteq U$, $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \cap V = \tilde{\phi}$. $\tilde{\tilde{x}} \in F \subseteq U$ iff $\tilde{\tilde{x}} \notin G \subseteq V$, since $U \cap V = \tilde{\phi}$ This implies $\tilde{\tilde{x}} \in \bar{G} \subseteq \bar{V}$, therefore $U \subseteq \bar{V}$ .Similarly If $\tilde{\tilde{x}} \notin F \subseteq V$, we get $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$ .Therefore (X,T) be satisfies WCN(iv).

WCN(iii) $\longrightarrow$ WCN(vi)

Let (X,T) be satisfies WCN(iii) if for each $x \in X$, $E \subseteq X$ and F,G are separation for E there exists $U, V \in WOX$ such that $\tilde{\tilde{x}} \in F \subseteq U$, $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$ $\tilde{\tilde{x}} \in F \subseteq U$ iff $x \in F_1 \subseteq U$, since $\langle x, F_1, F_2\rangle$ and $F_1 \cap F_2 = \phi$,this implies $x \notin F_2 \subseteq U$ iff $\tilde{\tilde{x}} \in F \subseteq U$ and $U \subseteq \bar{V}$ .Similarly If $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$, we get $\tilde{\tilde{x}} \notin G \subseteq V$ and $U \subseteq \bar{V}$

In general the converse of the diagram appears in Proposition 2-8 is not true in general. The following counter examples show the cases.

Example 2-9: Let $X = \{a,b\}$ and define $T = \{\tilde{\phi}, \tilde{X}, A, B, C, D\}$, where $A = \langle x,\phi,\{a\}\rangle$, $B = \langle x,\phi,\{b\}\rangle$, $C = \langle x,\phi,\phi\rangle$, $D = \langle x,\phi,\{a\}\rangle$ and WO(X) =T .Then (X,T) satisfies WCN(ii) because for each $x \in X$, $E \subseteq X$, $F = \langle x,\phi,\{a\}\rangle$, $G = \langle x,\phi,\{b\}\rangle$ are separation for E there exists $A, B \in WOX$ such that $\tilde{\tilde{a}} \in F \subseteq A, \tilde{\tilde{a}} \notin G \subseteq B$ and $A \cap B = \tilde{\phi}$ .Also $\tilde{b} \in G \subseteq B, \tilde{b} \notin F \subseteq A$ and $A \cap B = \tilde{\phi}$. But (X,T) is not satisfies CN(i),because there is no exist two weak open sets satisfies condition of WCN(i) .

Example 2-10: Let $X = \{a,b\}$ and define $T = \{\tilde{\phi}, \tilde{X}, A, B, C, D, E, F\}$,where $A = \langle x,\{a\},\phi\rangle$, $B = \langle x,\{b\},\{a\}\rangle$, $C = \langle x,\phi,\{a\}\rangle$ and WO(X) =T . Then (X,T) satisfies CN(iii),because for each $x \in X$, $E \subseteq X$ and $F = \langle x,\{b\},\{a\}\rangle$

$G=\langle x,\{a\},\phi\rangle$ are separation for E there exists $A,B\in WOX$ such that $\tilde{a}\in G\subseteq A,\tilde{a}\notin F\subseteq B$ and $B\subseteq\bar{A}$. Also $\tilde{b}\in F\subseteq B,\tilde{b}\notin G\subseteq A$ and $B\subseteq\bar{A}$. But (X,T) is not satisfies WCN(i) ,because there is no exist two weak open sets satisfies condition of WCN(i).

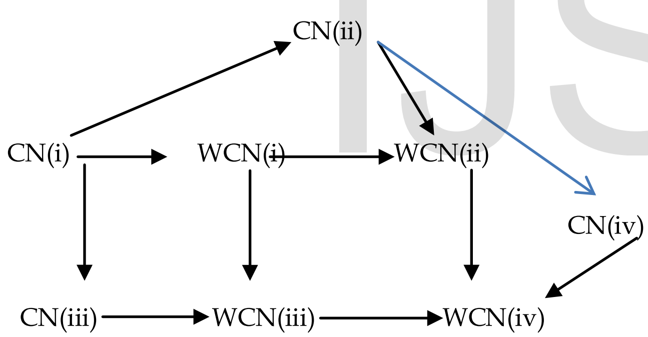
Example 2-11: Let $X=\{a,b\}$ and define $T=\{\tilde{\phi},\tilde{X},A,B,C,D\}$, where $A=\langle x,\phi,\{a\}\rangle$, $B=\langle x,\phi,\{b\}\rangle$, $C=\langle x,\phi,\phi\rangle$, $D=\langle x,\{a\},\phi\rangle$ and WO(X) =T . Then (X,T) satisfies WCN(vi)because for each x ∈ X , $E\subseteq X$, $F=\langle x,\phi,\{a\}\rangle$, $G=\langle x,\phi,\{b\}\rangle$ are separation for E there exists $A,B\in WOX$ such that $\tilde{\tilde{a}}\in F\subseteq A,\tilde{\tilde{a}}\notin G\subseteq B$ and $A\subseteq\bar{B}$. Also $\tilde{\tilde{b}}\in G\subseteq B,\tilde{\tilde{b}}\notin F\subseteq A$,and B ⊆ $\bar{A}$ .But (X,T) is not satisfies CN(iii)because there is no exist two weak open sets satisfies condition of WCN(iii). Also (X,T) is not satisfies WCN(ii).because there is no exist two weak open sets satisfies condition of WCN(ii).

3- The Relations Between Completely Normal and Weak Completely Normal

In this section we introduce the relation between completely normal and weak completely normal in intuitionistic topological spaces .

Proposition 3-1

Let (X,T) be an ITS, then the following implications are valid:



Proof : CN(i) $\longrightarrow$ WCN(i)

Let (X,T) be satisfies CN(i) if for each $x\in X$, $E\subseteq X$ and F,G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$ since every intuitionistic open set is intuitionistic weak open set , so that there exists $U,V\in WOX$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$ .Therefore (X,T) be satisfies WCN(i).

CN(ii) $\longrightarrow$ WCN(ii)

Let (X,T) be satisfies CN(ii) if for each $x\in X$, $E\subseteq X$ and F,G are separation there exists $U,V\in WOX$ such that $\tilde{\tilde{x}}\in F,F\subseteq U$, $\tilde{\tilde{x}}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$. since every intuitionistic open set is intuitionistic weak open set , so that there exists $U,V\in WOX$ such that $\tilde{\tilde{x}}\in F,F\subseteq U$, $\tilde{\tilde{x}}\notin G\subseteq V$ and $U\cap V=\tilde{\phi}$. Therefore (X,T) be satisfies WCN(ii)

CN(iii) $\longrightarrow$ WCN(iii)

Let (X,T) be satisfies CN(iii) if for each $x\in X$, $E\subseteq X$ and F , G are separation for E there exists $U,V\in T$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\subseteq\bar{V}$. since every open set is weak open set , so that there exists $U,V\in WOX$ such that $\tilde{x}\in F,F\subseteq U$, $\tilde{x}\notin G,G\subseteq V$ and $U\subseteq\bar{V}$ .Therefore (X,T) be satisfies WCN(iii).

Remark 3-2 : by transitive:

CN(i) $\longrightarrow$ WCN(ii) , CN(i) $\longrightarrow$ WCN(iii)
WCN(i) $\longrightarrow$ WCN(iv) ,CN(ii) $\longrightarrow$ WCN(iv)
CN(iii) $\longrightarrow$ WCN(iv).

In general the converse of the diagram appears in Proposition 3-1 is not true in general . The following counter examples show the cases .

Example 3-3: Let $X=\{a,b\}$ and define $T=\{\tilde{\phi},\tilde{X},A,B\}$,where $A=\langle x,\phi,\{a\}\rangle$, $B=\langle x,\phi,\phi\rangle$, and $WO(X)=T\cup\{C,D\}$ where $C=\langle x,\{a\},\{b\}\rangle$, $D=\langle x,\{b\},\{a\}\rangle$.Then (X,T) satisfies WCN(i),because for each $x\in X$, E⊆ X and $F=\langle x,\{a\},\{b\}\rangle$, $G=\langle x,\{b\},\{a\}\rangle$. are separation for E there exists $A,B\in WOX$ such that $\tilde{a}\in F\subseteq C,\tilde{a}\notin G\subseteq D$ and $C\cap D=\tilde{\phi}$. Also $\tilde{b}\in G\subseteq D,\tilde{b}\notin F\subseteq C$ and $C\cap D=\tilde{\phi}$. But (X,T) is not satisfies CN(i) ,because there is no exist two open sets satisfies condition of CN(i) .

Example 3-4: Let $X=\{a,b\}$ and define $T=\{\tilde{\phi},\tilde{X},A,B\}$,where $A=\langle x,\phi,\{a\}\rangle$, $B=\langle x,\phi,\phi\rangle$ and $WO(X)=T\cup\{C,D,E\}$ where $C=\langle x,\phi,\{b\}\rangle$, $D=\langle x,\{b\},\{a\}\rangle$, $E=\langle x,\{a\},\{b\}\rangle$.Then (X,T) satisfies WCN(ii) ,because for each $x\in X$, $E\subseteq X$ and $F=\langle x,\{a\},\{b\}\rangle$, $D=\langle x,\{b\},\{a\}\rangle$. are separation for E there exists $A,B\in WOX$ such that $\tilde{\tilde{a}}\in F\subseteq E,\tilde{\tilde{a}}\notin G\subseteq D$ and. $D\cap E=\tilde{\phi}$ Also $\tilde{\tilde{b}}\in G\subseteq D,\tilde{\tilde{b}}\notin F\subseteq E$,and $D\cap E=\tilde{\phi}$ .But (X,T) is not satisfies CN(ii)because there is no exist two open sets satisfies condition of CN(ii) .

Example 3-5: Let $X=\{a,b\}$ and define $T=\{\tilde{\phi},\tilde{X},A\}$, where $A=\langle x,\{a\},\{b\}\rangle$, and $WO(X)=T\cup\{B,C,D,E,F,H\}$ where $B=\langle x,\{b\},\{a\}\rangle$, $C=\langle x,\phi,\{b\}\rangle$ $D=\langle x,\phi,\{a\}\rangle$, $E=\langle x,\{b\},\phi\rangle$, $F=\langle x,\{a\},\phi\rangle$, $H=\langle x,\phi,\phi\rangle$.Then (X,T) satisfies WCN(iii), because for each $x\in X$, $E\subseteq X$ and $F=\langle x,\{a\},\{b\}\rangle$, $G=\langle x,\{b\},\{a\}\rangle$,are separation for E there exists $A,B\in WOX$ such that $\tilde{a}\in F\subseteq A,\tilde{a}\notin G\subseteq B$ and $A\subseteq\bar{B}$. Also $\tilde{b}\in G\subseteq B,\tilde{b}\notin F\subseteq A$ and $B\subseteq\bar{A}$. But (X,T) is not satisfies CN(iii) because there is no exist two open sets satisfies condition of CN(iii) .

Example 3-6: Let $X=\{a,b\}$ and define $T=\{\tilde{\phi},\tilde{X},A,B,C\}$, where $A=\langle x,\phi,\{a\}\rangle$, $B=\langle x,\phi,\phi\rangle$, $C=\langle x,\{a\},\phi\rangle$ and $WO(X)=T\cup\{D,E\}$ where, $D=\langle x,\{b\},\{a\}\rangle$, $E=\langle x,\{a\},\{b\}\rangle$. Then (X,T) satisfies WCN(iv),because for each $x\in X$, E ⊆ X and $F=\langle x,\phi,\{b\}\rangle$ $G=\langle x,\phi,\{a\}\rangle$,are separation for E there exists $A, B \in WOX$ such that $\tilde{\tilde{a}} \in G \subseteq A, \tilde{\tilde{a}} \notin F \subseteq C$, and $A \subseteq \overline{C}$. Also $\tilde{\tilde{b}} \in F \subseteq C, \tilde{\tilde{b}} \notin G \subseteq A$ and $C \subseteq \overline{A}$. But (X, T) is not satisfies CN(iv), because there is no exist two open sets satisfies condition of CN(iv).

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